ФИЗИКО-МАТЕМАТИЧЕСКИЕ НАУКИ

IS IT POSSIBLE TO SOLVE THE ANGLE TRISECTION PROBLEM WITH COMPASS-STRAIGHTEDGE CONSTRUCTION?

(short note)

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If you pose the question given in the title of this note you will listen a negative answer. In the Google searcher you will receive about 6 million results. It means to find something new in the problem formulated by Greek mathematician is useless. This problem alongside with the circle squaring is considered as *undecidable* problem.

As stated by Pierre Laurent Wantzel (1837), the solution of the angle trisection problem corresponds to an implicit solution of the cubic equation $x^3 - 3x - 1 = 0$, which is algebraically irreducible, and so this statement is equivalent to the geometric solution of the angle trisection problem. A classic geometric solution was given in paper with references therein [1].

Nevertheless, in this short note we want to demonstrate the simplest solution of the angle trisection problem (ATP) with the help of the compass-straightedge construction, adding a pencil and sheet of paper. Two last items are needed only for demonstration purposes.

The simplest solution will be based on the following geometric sums

$$S_N(+x) = x \cdot (1 + x + x^2 + \ldots + x^{N-1}) = \left(\frac{x}{1-x}\right) \cdot (1 - x^N),$$
$$S_N(-x) = x \cdot (1 - x + x^2 + \ldots + (-x)^{N-1}) = \left(\frac{x}{1+x}\right) \cdot (1 - (-x)^N). \quad (1)$$

In order to receive the desired solution, we put $x = 1/4$and present the initial sum in the form

$$\varphi_0 \cdot S_N\left(\frac{1}{4}\right) = \varphi_0 \cdot \left[\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots + \left(\frac{1}{4}\right)^N\right] \equiv \frac{\varphi_0}{3}\left(1 - \varepsilon_N\left(\frac{1}{4}\right)\right),$$
$$\varepsilon_N\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^N. \quad (2)$$

Here φ_0 is the initial angle that is needed to be trisected, $\varepsilon_N(1/4)$ – is a small correction. This sum is converged to the value $\varphi_0/3$ at $N >>1$. Having a simple ruler and pencil one can pose the following question: how many terms it is necessary to keep in (2) for receiving the trisected angle with the given accuracy? We suppose that the pencil-point has 1 mm (or a bit less) and the trace left by the pencil on a sheet of paper has the same length. Therefore, we have 4^N=1000 that gives N value located between 4 and 5. It implies that in the normal conditions having the A4 format sheet of paper, normal sharp pencil or ball pen one can keep only 4 and 5 terms in (2) for solving the ATP with the given accuracy $\varepsilon_4(1/4) = (1/4)^4 \cdot 100\% \cong 0.4\%$. If we keep the next term then one can receive $\varepsilon_5(1/4) \cdot 100\% \cong 0.098\%$. Therefore, supposing that our divider compass can provide the infinite division of any angle on two equal parts ($N >>1$) we showed how to solve the ATP with the given accuracy.

Attentive analysis show that putting $x=2^{-q}$ (q=2, 3,…) one can provide more interesting solutions and solve the angle n-section solution for some odd numbers as n=5,7,9,…But before it is interesting to note also that ATP admits another solution. For this aim we consider the second sum in (1).

$$\varphi_0 \cdot S_N\left(-\frac{1}{2}\right) = \varphi_0 \cdot \left[\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} \ldots + \left(-\frac{1}{2}\right)^{N+1}\right] =$$
$$\frac{\varphi_0}{3}\left(1 - \varepsilon_N\left(\frac{1}{2}\right)\right) \overset{N>>1}{\cong} \frac{\varphi_0}{3}, \varepsilon_N\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^N. \quad (3)$$

However, this solution is a poor choice and can be omitted because of the slow convergence of the sum $S_N(-1/2)$ to the value $\varphi_0/3$.

It is convenient to present other possible angle n-section solutions in the form of the table when any chosen angle admits division on two equal parts with the help of a straightedge ruler and divider compass.

Table showing a possible angle n-section realized with the help of a ruler and divider.

q-value $x=2^{-q}$	$\frac{\varphi_0}{2^q-1}$	$\frac{\varphi_0}{2^q+1}$	Value of the error $E(q,N)=(2^{-qN})\cdot 100\%$
2	$\frac{\varphi_0}{3}$	$\frac{\varphi_0}{5}$	$E(2,4)=3.91\cdot10^{-1}(\%)$ $E(2,5)=9.77\cdot10^{-2}(\%)$
3	$\frac{\varphi_0}{7}$	$\frac{\varphi_0}{9}$	$E(3,3)=1.95\cdot10^{-1}(\%)$ $E(3,4)=2.44\cdot10^{-2}(\%)$
4	$\frac{\varphi_0}{15}$	$\frac{\varphi_0}{17}$	$E(4,3)=2.44\cdot10^{-2}(\%)$ $E(4,4)=1.53\cdot10^{-3}(\%)$

As it follows from this table with increasing of the *n-th* angle-sected value of an admissible error value is decreasing. The author things that this simplest solution put a final point in solution of the ATP attracting the attention of many mathematicians.

References

[1] C. Rediske, The Trisection of an Arbitrary Angle: A Classical Geometric Solution, J. of Advances in Mathematics (2018) pp. 7640-7669. DOI: 10.24297/jam.v14i2.7402.

К ВОПРОСУ НЕЙТРИННОГО ИЗЛУЧЕНИЯ В РАСШИРЯЮЩЕЙСЯ С ОХЛАЖДЕНИЕМ ВСЕЛЕННОЙ

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ON THE QUESTION OF NEUTRINO RADIATION IN THE UNIVERSE EXPANDING WITH COOLING

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Аннотация. Рассмотрена однородная модель Вселенной в виде газовой смеси из фотонов, барионов и нейтрино. По мере анализа изучаемой физической системы воспроизведена математическая структура для описания ее космологической эволюции. Выполнена оценка средней величины энергии реликтового нейтрино, которая совпала с известной оценкой профессора О. Лахав (2002 г.) по порядку величины. Приведены аргументы в пользу первичного термоядерного взрыва у истока расширения Вселенной на планковском масштабе времени.

Abstract. A homogeneous model of the Universe in the form of a gas mixture of photons, baryons and neutrinos is considered. As the physical system under study is analyzed, the mathematical structure for describing its cosmological evolution is reproduced. The average value of the relic neutrino energy was estimated, which coincided with the well-known estimate of Professor O. Lahav (2002) in order of magnitude. Arguments are given in favor of a primary thermonuclear explosion at the source of the expansion of the Universe on the Planck time scale.

Ключевые слова: модель расширяющейся Вселенной, реликтовое излучение, планковские величины, закон Стефана – Больцмана, объемная плотность энергии нейтрино, масса реликтового нейтрино.

Keywords: model of the expanding Universe, relic radiation, Planck quantities, Stefan – Boltzmann law, volume density of neutrino energy, mass of the relic neutrino.

«Структура математического описания выявляется
по мере анализа физической системы»
П. Девис [1, с. 260]

Как известно, космология, которая изучает свойства Вселенной в целом, – одна из немногих естественных наук, где присутствует эволюция в явном виде. Изучение эволюции Вселенной осложнено тем, что её динамика отнюдь не представляет собой нечто непосредственно наблюдаемое. По этой причине объект исследования может быть дан как целое лишь